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Generation of zonal flows by interchange modes in a plasma

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Abstract. The generation of zonal flows by flute-like interchange modes in a nonuniform magnetoplasma is considered. The guiding center particle drifts are then used to derive a system of coupled mode equations. The latter are Fourier analyzed to obtain a nonlinear dispersion relation, which exhibits the excitation of zonal flows by the ponderomotive force of the interchange modes. The growth rate of the parametrically driven zonal flows is obtained.

PACS. 52.25.Vy Impurities in plasmas – 52.35.Mw Nonlinear phenomena: waves, wave propagation, and other interactions (including parametric effects, mode coupling, ponderomotive effects, etc.) – 52.35.Ra Plasma turbulence

Recently, several authors [1,2] investigated the excitation of zonal flows [3–6] and streamers by low-frequency (in comparison with the ion gyrofrequency ω_{ci}) interchange modes in plasmas containing unfavorable curvatures of the magnetic field with anti-parallel density gradients, which are common in laboratories [7–11] and in space [15]. Das *et al.* [1] presented detailed numerical and computational studies of zonal flows and streamers associated with magnetic curvature driven Rayleigh-Taylor (R-T) instabilities, while Andrushchenko *et al.* [2] considered the generation of large scale zonal flows by incoherent R-T modes.

A zonal flow is defined as an azimuthally symmetric two-dimensional perturbation with a finite radial scale $2\pi/q_r$ which is significantly larger than the scale $2\pi/k_r$ of its drivers (*e.g.* interchange modes), where q_r and k_r are the radial wave numbers of long wavelength zonal flows and short wavelength drivers, respectively. On the other hand, streamers have short poloidal extent and are radially elongated structures. Both zonal flows and streamers, which are experimentally observed [7–11] in tokamaks, pinches and stellarators, are supposed to be excited by drift and other waves [4,12,13], and they seem to play a detrimental role in regulating turbulent transport [14].

In this paper, we consider the nonlinear excitation of zonal flows by marginally stable coherent R-T or interchange modes in a nonuniform magnetoplasma containing inhomogeneous magnetic field ($\partial B_0/\partial r$) and equilibrium density ($\partial n_0/\partial r$) gradients in the radial direction, where $\hat{\mathbf{z}}B_0$ is the equilibrium magnetic field and n_0 is the unperturbed plasma number density. In this nonuniform magnetic field, the ions have an equilibrium azimuthal drift $\mathbf{V}_g$ due to the gravity force. The interchange modes are two-dimensional electrostatic disturbances containing significant density fluctuations, in contrast to the zonal flows which are purely damped (due to the ion gyroviscosity) flute-like convective cells accompanying insignificant density perturbations and with zero azimuthal wavenumber.

The perpendicular components of the electron and ion fluid velocities in the presence of nonlinearly coupled low-frequency (in comparison with $\omega_{ci} = eB_0/m_ic$, where e is the magnitude of the electron charge, m_i is the ion mass and c is the speed of light in vacuum) and long wavelength (in comparison with the ion gyroradius $\rho_i = v_{ti}/\omega_{ci}$, where v_{ti} is the ion thermal speed) interchange modes and zonal flows are, respectively,

$$\mathbf{v}_{e\perp}^{ic} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \nabla\phi - \frac{cT_e}{eB_0n_0}\hat{\mathbf{z}} \times \nabla n_{e1}^{ic}, \tag{1}$$

and

$$\begin{aligned}\mathbf{v}_{i\perp}^{ic} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \nabla\phi + \frac{cT_i}{eB_0n_0}\hat{\mathbf{z}} \times \nabla n_{i1}^{ic} \\ - \frac{c}{B_0\omega_{ci}}\left(\partial_t + \mathbf{V}_g \cdot \nabla + \mathbf{V}_* \cdot \nabla - \mu_i\nabla_\perp^2\right)\nabla_\perp\phi \\ - \frac{c^2}{B_0^2\omega_{ci}}\left[(\hat{\mathbf{z}} \times \nabla\varphi \cdot \nabla)\nabla_\perp\psi + (\hat{\mathbf{z}} \times \nabla\psi \cdot \nabla)\nabla_\perp\varphi\right],\end{aligned} \tag{2}$$

$$\mathbf{v}_{e\perp}^{z} \approx \frac{c}{B_0}\hat{\mathbf{z}} \times \nabla\psi \tag{3}$$

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and

$$\mathbf{v}_{i\perp}^{z} \approx \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\psi - \frac{c}{B_0\omega_{ci}}\Big[(\partial_t - \mu_i\nabla_\perp^2)\nabla_\perp\psi + \frac{c}{B_0}\left\langle(\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla)\nabla_\perp\phi\right\rangle\Big], \quad (4)$$

where ϕ and ψ are the electrostatic potentials of the interchange modes and zonal flows, respectively, $\varphi = \phi + T_i n_{i1}^{ic}/en_0$, T_j is the temperature, $\mathbf{V}_g = \hat{\theta}V_g \equiv \hat{\theta}g/\omega_{ci}$ represents the equilibrium azimuthal ion drift caused by the gravity $g \equiv v_{ti}^2/R$, R is the radius of curvature of the nonuniform external magnetic field, $\mathbf{V}_* = (cT_i/eB_0n_0)\hat{\mathbf{z}}\times\nabla n_0$ is the unperturbed ion diamagnetic drift, $\mu_i = (3/10)\nu_i\rho_i^2$ represents the ion gyroviscosity, and ν_i is the ion–ion collision frequency. The number density perturbations are denoted by n_{j1}. The electron and ion motions are assumed two-dimensional. The superscripts ic and z represent the quantities associated with the interchange modes and zonal flows, respectively. The angular brackets denote averaging over one period of the interchange modes.

Substituting (1) and (2) into the electron and ion continuity equations, subtracting the resulting equations and use the quasineutrality approximation $n_{i1}^{ic} \approx n_{e1}^{ic}$, which holds for $\omega_{pi} \gg \omega_{ci}$, where $\omega_{pi} = (4\pi n_0 e^2/m_i)^{1/2}$ is the ion plasma frequency, we obtain the equation for the interchange modes in the presence of zonal flows. We have

$$\left(\partial_t + V_g\partial_\theta + \mathbf{V}_*\cdot\nabla - \mu_i\nabla_\perp^2\right)\nabla_\perp^2\phi - \frac{B_0\omega_{ci}}{cn_0}V_g\partial_\theta n_{e1}^{ic} + \frac{c}{B_0}\Big\{(\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla)\nabla_\perp^2\psi + (\hat{\mathbf{z}}\times\nabla\psi\cdot\nabla)\nabla_\perp^2\phi + \frac{T_i}{en_0}\nabla\cdot\left[\left(\hat{\mathbf{z}}\times\nabla n_{i1}^{ic}\right)\cdot\nabla\nabla_\perp\psi + (\hat{\mathbf{z}}\times\nabla\psi)\cdot\nabla\nabla_\perp n_{i1}^{ic}\right]\Big\} = 0. \quad (5)$$

We note that the divergence of $(c/B_0)\hat{\mathbf{z}}\times\nabla\phi\cdot(n_{i1}^z - n_{e1}^z)$ is much smaller (since $\omega_{pi} \gg \omega_{ci}$) than the divergence of the nonlinear ion polarization terms that are retained in (5), where $n_{i1}^z - n_{e1}^z = (1/4\pi e)\nabla_\perp^2\psi$ represents the charge separation effect associated with zonal flows which have $e\psi/T_e \gg (n_{i1}^z/n_0) \equiv (c/B_0\omega_{ci})\nabla_\perp^2\psi$. Here $\rho_s^2\nabla_\perp^2\psi \ll \psi$, $\rho_s = c_s/\omega_{ci}$ is the ion gyroradius at the electron temperature, and $c_s = (T_e/m_i)^{1/2}$ is the ion sound speed. The electron density perturbation n_{e1}^{ic} associated with interchange modes is obtained from

$$\partial_t n_{e1}^{ic} - c\hat{\mathbf{z}}\times\nabla(n_0/B_0)\cdot\nabla\phi + \frac{c}{B_0}\hat{\mathbf{z}}\times\nabla\psi\cdot\nabla n_{e1}^{ic} = 0, \quad (6)$$

where the last terms in the left-hand side of (5) and (6) are due to the coupling between interchange modes and zonal flows on account of the nonlinear ion polarization drift and the $\hat{\mathbf{z}}\times\nabla\psi$ zonal flow coupling with the interchange mode density fluctuations, respectively. In the absence of the zonal flows, *viz.* without the ψ-terms, equations (5, 6) can be combined and Fourier analyzed to obtain the frequency spectrum of stable interchange modes for $g\partial\ln(n_0/B_0)/\partial r > 0$, which is of our interest here. The interchange mode frequency Ω is then determined from $\Omega[\Omega - K_\theta(V_g + U_*) + \mathrm{i}\mu_i K_\perp^2] + \omega_{ci}K_{nb}V_gK_\theta^2/K_\perp^2 = 0$, where $U_* = (cT_i/eB_0n_0)\partial n_0/\partial r$, $K_{nb} = \partial\ln(n_0/B_0)/\partial r$ and K_θ and $K_\perp$ are the theta and radial components of the wave vector $\mathbf{K}$, respectively.

On the other hand, the equation for zonal flows is obtained by inserting (3) and (4) into the charge current density equation and using Poisson's equation. We have

$$\left(\partial_t - \mu_i\nabla_\perp^2\right)\nabla_\perp^2\psi + \frac{c}{B_0}\left\langle(\hat{\mathbf{z}}\times\nabla\phi\cdot\nabla)\nabla_\perp^2\phi\right\rangle = 0, \quad (7)$$

where the third term in the left-hand side of (7) represents the ponderomotive force of the interchange modes. Equations (5–7) are the desired equations for studying the excitation of zonal flows by large amplitude interchange modes which are marginally stable.

The nonlinear interactions between a finite amplitude interchange pump mode $(\omega_0, \mathbf{k}_0)$ and zonal flows $(\omega, \mathbf{k})$ excite upper and lower interchange sidebands $(\omega_\pm, \mathbf{k}_\pm)$. Thus, we decompose the interchange mode potential and the density perturbation as

$$\phi = \phi_{0+}\exp(-\mathrm{i}\omega_0 t + \mathrm{i}\mathbf{k}_0\cdot\mathbf{r}) + \phi_{0-}\exp(\mathrm{i}\omega_0 t - \mathrm{i}\mathbf{k}_0\cdot\mathbf{r}) + \sum_{+,-}\phi_\pm\exp(-\mathrm{i}\omega_\pm t + \mathrm{i}\mathbf{k}_\pm\cdot\mathbf{r}), \quad (8)$$

and

$$n_{e1}^{ic} = n_{10+}\exp(-\mathrm{i}\omega_0 t + \mathrm{i}\mathbf{k}_0\cdot\mathbf{r}) + n_{10-}\exp(\mathrm{i}\omega_0 t - \mathrm{i}\mathbf{k}_0\cdot\mathbf{r}) + \sum_{+,-}n_{1\pm}\exp(-\mathrm{i}\omega_\pm t + \mathrm{i}\mathbf{k}_\pm\cdot\mathbf{r}), \quad (9)$$

where $\omega_\pm = \omega \pm \omega_0$ and $\mathbf{k}_\pm = \mathbf{k} \pm \mathbf{k}_0$ are the frequencies and wave vectors of the sidebands, and the superscript 0 ($\pm$) stands for the pump (sidebands).

Inserting equations (8, 9) into equations (5, 6) and Fourier analyzing we obtain, respectively,

$$D_\pm\phi_\pm - \frac{B_0\omega_{ci}}{c}V_g\frac{k_{\theta\pm}}{k_{\perp\pm}^2}\frac{n_{1\pm}}{n_0} = \pm\,\mathrm{i}\frac{cp}{B_0}\frac{\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{k}}{k_{\perp\pm}^2}(k_\perp^2 - k_{\perp 0}^2)\phi_{0\pm}\hat{\psi}, \quad (10)$$

and

$$\omega_\pm n_{1\pm} + c\hat{\mathbf{z}}\times\nabla(n_0/B_0)\cdot\mathbf{k}_\pm\phi_\pm \mp \mathrm{i}\frac{c}{B_0}\hat{\mathbf{z}}\times\mathbf{k}\cdot\mathbf{k}_0 n_{10\pm}\hat{\psi}, \quad (11)$$

with

$$n_{10\pm} = -\frac{c}{\omega_0}\mathbf{k}_0\cdot\hat{\mathbf{z}}\times\nabla(n_0/B_0)\phi_{0\pm}, \quad (12)$$

where $D_\pm = \omega_\pm + \mathrm{i}\Gamma_\pm - \omega_* - \omega_{g\pm}$, $\Gamma_\pm = \mu_i k_{\perp\pm}^2$, $\omega_* = \mathbf{k}_\pm\cdot\mathbf{V}_*$ is the ion drift wave frequency, $\omega_{g\pm} = k_{\theta\pm}V_g$, $p = 1 + k_{\theta 0}K_n\rho_i^2\omega_{ci}/\omega_0$, and $K_n = n_0^{-1}\partial n_0/\partial r$. Furthermore, $k_{\theta 0}$ and $\mathbf{k}_\perp$ are the θ and perpendicular components of the wave vector, respectively. In deriving (10) and (11) we have introduced $\psi = \hat{\psi}\exp(-\mathrm{i}\omega t + \mathrm{i}\mathbf{k}\cdot\mathbf{r})$ and matched the phasors.

Inserting (8) into (7) and Fourier analyzing, we have

$$(\omega + \mathrm{i}\Gamma_z)\hat{\psi} = \mathrm{i}\frac{c}{B_0}\frac{\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{k}}{k_\perp^2}\left(K_-^2\phi_{0+}\phi_- - K_+^2\phi_{0-}\phi_+\right), \quad (13)$$

where $K_\pm^2 = k_{\perp\pm}^2 - k_0^2$ and $\Gamma_z = \mu_i k_\perp^2$.

Combining (10), (11) and (13) we obtain

$$\omega + \mathrm{i}\Gamma_z = \frac{pc^2|\phi_0|^2}{B_0^2}\frac{|\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{k}|^2}{k_\perp^2}\sum_{+,-}\frac{K_\pm^2\mathcal{K}_\pm^2}{k_{\perp\pm}^2\mathcal{D}_\pm}, \quad (14)$$

where $|\phi_0|^2 = \phi_{0+}\phi_{0-}$, $\mathcal{K}_\pm^2 = k_\perp^2 - k_{\perp 0}^2 + (k_{\theta\pm}V_g\omega_{ci}/\omega_0\omega_\pm)\,\mathbf{k}_0\cdot\hat{\mathbf{z}}\times\ln(n_0/B_0)$, and $\mathcal{D}_\pm = D_\pm + (\omega_{ci}/\omega_\pm)k_{\theta\pm}V_g\mathbf{k}_\pm\cdot\hat{\mathbf{z}}\times\nabla\ln(n_0/B_0)/k_{\perp\pm}^2$.

For $|\omega| \gg \Gamma_z$ and $k_{\perp 0} \gg k_\perp$ we obtain from (14) the growth rate of a purely growing ($\omega = \mathrm{i}\gamma$) instability for $\mathbf{k}_0\cdot\mathbf{k} > 0$. We have

$$\gamma = \sqrt{2p}\left|\frac{c\phi_0}{B_0}\frac{\hat{\mathbf{z}}\times\mathbf{k}_0\cdot\mathbf{k}}{k_\perp}\right||\mathbf{k}_0\cdot\mathbf{k}_\perp|^{1/2}(1+\alpha)^{1/2}. \quad (15)$$

where $\alpha = (k_{\theta+}V_g\omega_{ci}/2\omega_0^2)|\mathbf{k}_0\cdot\hat{\mathbf{z}}\times\nabla\ln(n_0/B_0)/\mathbf{k}_0\cdot\mathbf{k}_\perp|$. Equation (15) predicts that the growth rate of the purely growing mode is directly proportional to the pump wave electric field $\mathbf{k}_0|\phi_0|$.

Let us estimate the growth rate for a typical laboratory plasma [10,11] with $B_0 \sim 3$ kG, $T_i \sim 5-10$ eV, $\rho_i \sim 0.05-0.1$ cm, $L_{nb} = 50$ cm, and $\phi_0 \sim 1$ V, where $L_{nb}^{-1} = \partial\ln(n_0/B_0)/\partial r$. For these parameters the growth time $(1/\gamma)$ is roughly ten microseconds when $k_{\perp 0} \sim 1$ cm^{-1} and $k_\perp = 0.1$ cm^{-1}.

To summarize, we have considered the nonlinear interaction between marginally stable coherent interchange modes and zonal flows in a nonuniform magnetoplasma that contains a density gradient and an unfavorable magnetic curvature. We have used a two-fluid model to derive the governing equations for two-dimensional interchange modes and zonal flows, taking into account their nonlinear couplings. The coupled mode equations have then been Fourier analyzed following standard techniques [4,12,13] to derive a general dispersion relation for parametric processes. The dispersion relation reveals a purely growing instability due to which zonal flows are excited. An explicit expression for the growth rate is presented. The results of the present investigation offers a plausible mechanism for exciting sheared (zonal) flows in the presence of interchange mode turbulence that may exist in low-temperature plasmas such as those in the Earth's ionosphere [15] as well as in tokamak edges [10] and currentless toroidal plasma devices [16]. For parameters representative of tokamak edges [10,11], the growth time for the zonal flow excitation is roughly ten microseconds. The nonlinearly excited zonal flows may, in turn, control the transport processes in those regions of the plasmas where nonuniform magnetic fields and density gradients are inherently present.

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References

1. A. Das, A. Sen, S. Mahajan, P. Kaw, Phys. Plasmas **8**, 5104 (2001)
2. Zh.N. Andrushchenko *et al.*, Phys. Scr. **66**, 8 (2002)
3. H. Okuda, J.M. Dawson, Phys. Fluids **16**, 480 (1973)
4. P.K. Shukla *et al.*, Phys. Rev. A **23**, 321 (1981); P.K. Shukla, L. Stenflo, Eur. Phys. J. D **20**, 103 (2002)
5. A. Hasegawa, Adv. Phys. **1**, 234 (1985)
6. T. Soomere, Phys. Rev. Lett. **75**, 2440 (1995)
7. Ch.P. Ritz, H. Lin, T.L. Rhodes, A.J. Wooton, Phys. Rev. Lett. **65**, 2543 (1990)
8. V. Antoni *et al.*, Phys. Rev. Lett. **79**, 4814 (1997)
9. C. Hidalgo *et al.*, Phys. Rev. Lett. **83**, 2203 (1999)
10. S. Coda, M. Porkolab, K.H. Burrell, Phys. Rev. Lett. **86**, 4835 (2001)
11. R.A. Moyer *et al.*, Phys. Rev. Lett. **87**, 135001 (2001); G.R. Tynan *et al.*, Phys. Rev. Lett. **88**, 2691 (2001)
12. L. Chen, Z. Lin, R. White, Phys. Plasmas **7**, 3129 (2000)
13. P.K. Shukla, L. Stenflo, Phys. Plasmas **9**, 3636 (2002)
14. J. Lin *et al.*, Science **281**, 1835 (1998); T.S. Hahm *et al.*, Phys. Plasmas **6**, 992 (1999)
15. A.B. Hassam, W. Hall, J.D. Huba, J. Keskinen, J. Geophys. Res. **91**, 1313 (1986)
16. A. Das *et al.*, Phys. Plasmas **4**, 1018 (1997)